**MATRIC NO:19/SCI01/090**

**NAME:ABEGUNDE OLAKUNLE OLUWADUNSIN**

1)

**Definition**

A *linear transformation* is a transformation T:Rn→Rm satisfying

T(u+v)=T(u)+T(v)T(cu)=cT(u)

for all vectors u,v in Rn and all scalars c.

**Facts about linear transformations**

*Let T:Rn→Rm be a linear transformation. Then:*

*T(0)=0.*

*For any vectors v1,v2,...,vk in Rn and scalars c1,c2,...,ck, we have*

*TAc1v1+c2v2+···+ckvkB=c1T(v1)+c2T(v2)+···+ckT(vk).*

**Examples**

**i)**

Define T:R→R by T(x)=x+1. Is T a linear transformation?

Solution

We have T(0)=0+1=1. Since any linear transformation necessarily takes zero to zero by the above important note, we conclude that T is *not* linear (even though its graph is a line).

*Note:* in this case, it was not necessary to check explicitly that T does not satisfy both defining properties: since T(0)=0 is a consequence of these properties, at least one of them must not be satisfied. (In fact, this T satisfies neither.)

**ii)**

Define T:R2→R2 by T(x)=1.5x. Verify that T is linear.

Solution

We have to check the defining properties for *all* vectors u,v and *all* scalars c. In other words, we have to treat u,v, and c as *unknowns*. The only thing we are allowed to use is the definition of T.

T(u+v)=1.5(u+v)=1.5u+1.5v=T(u)+T(v)

Since T satisfies both defining properties, T is linear.

*Note:* we know from this example in Section 3.1 that T is a matrix transformation: in fact,

.

Since a matrix transformation is a linear transformation, this is another proof that T is linear.

**iii)**

Define T:R2→R3 by the formula

T=.

Verify that T is linear.

Solution

We have to check the defining properties for *all* vectors u,v and *all* scalars c. In other words, we have to treat u,v, and c as *unknowns*; the only thing we are allowed to use is the definition of T. Since T is defined in terms of the coordinates of u,v, we need to give those names as well; say u= and v=. For the first property, we have

For the second property,

Since T satisfies the defining properties, T is a linear transformation.

*Note:* we will see in this example below that

Hence T is in fact a matrix transformation.

**iv)**

Verify that the following transformations from R2 to R2 are not linear:

T1

Solution

In order to verify that a transformation T is *not* linear, we have to show that T does not satisfy *at least one* of the two defining properties. For the first, the negation of the statement “T(u+v)=T(u)+T(v) for all vectors u,v” is “there exists at least one pair of vectors u,v such that T(u+v)A=T(u)+T(v).” In other words, it suffices to find *one example* of a pair of vectors u,v such that T(u+v)A=T(u)+T(v). Likewise, for the second, the negation of the statement “T(cu)=cT(u) for all vectors u and all scalars c” is “there exists some vector u and some scalar c such that T(cu)A=cT(u).” In other words, it suffices to find *one* vector u and *one* scalar c such that T(cu)A=cT(u).

For the first transformation, we note that

but that

Therefore, this transformation does not satisfy the second property.

For the second transformation, we note that

but that

Therefore, this transformation does not satisfy the second property.

For the third transformation, we observe that

Since T3 does not take the zero vector to the zero vector, it cannot be linear.

**v)**

Define T:C3→C2T:C3→C2 by describing the output of the function for a generic input with the

and check the two defining properties.

T(x)+T(y)

And

T is a linear transformation.